Probabilistic broadcasting of mixed states

2009 J. Phys. A: Math. Theor. 42 175302

(http://iopscience.iop.org/1751-8121/42/17/175302)

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **42** (2009) 175302 (13pp) doi:10.1088/1751-8113/42/17/175302

Probabilistic broadcasting of mixed states

Lvjun Li[1], **Daowen Qiu**[2], **Lvzhou Li**[1], **Lihua Wu**[1] **and Xiangfu Zou**[1]

[1] Department of Computer Science, Zhongshan University, Guangzhou 510275, People's Republic of China

[2] SQIG-Instituto de Telecomunições, IST, TULisbon, Av. Rovisco Pais 1049-001, Lisbon, Portugal

E-mail: issqdw@mail.sysu.edu.cn

Received 24 October 2008, in final form 19 February 2009

Published 7 April 2009

Online at stacks.iop.org/JPhysA/42/175302

Abstract

It is well known that the non-broadcasting theorem proved by Barnum *et al* is a fundamental principle of quantum communication. As we are aware, *optimal broadcasting* (OB) is the only method to broadcast noncommuting mixed states approximately. In this paper, motivated by the probabilistic cloning of quantum states proposed by Duan and Guo, we propose a new way for broadcasting noncommuting mixed states—*probabilistic broadcasting* (PB), and we present a sufficient condition for PB of mixed states. To a certain extent, we generalize the probabilistic cloning theorem from pure states to mixed states, and in particular, we generalize the non-broadcasting theorem, since the case that commuting mixed states can be exactly broadcast can be thought of as a special instance of PB where the success ratio is 1. Moreover, we discuss *probabilistic local broadcasting* (PLB) of separable bipartite states.

PACS numbers: 03.67.−a, 03.65.Ud

1. Introduction

Quantum broadcasting [1] means that the marginal density operator is the same as the density operator of the state to be broadcast. More specifically, Alice has an N-dimensional system A, and the initial state is secretly chosen from the set $\{\rho_i \mid i = 1, 2\}$ to be broadcast; Bob also has an N-dimensional system B whose initial state is a blank state Σ. Suppose that the initial state of the composite system AB is $\rho_i \otimes \Sigma$. Then, the broadcasting problem is to investigate whether there exists an operation ξ acting on the composite system AB such that the following equation holds

$$\xi(\rho_i \otimes \Sigma) = \widetilde{\rho}_i, \qquad i = 1, 2, \tag{1}$$

where $\widetilde{\rho}_i$ denotes a state of the composite system AB satisfying $\mathrm{tr}_A(\widetilde{\rho}_i) = \mathrm{tr}_B(\widetilde{\rho}_i) = \rho_i$, and here tr_A and tr_B denote partial traces over the subsystems A and B, respectively. We can consider quantum cloning as a special case of broadcasting that $\widetilde{\rho}_i = \rho_i \otimes \rho_i$ in equation (1). In this manner, if we have a machine to clone $\{\rho_i\}$, then we have a machine to broadcast $\{\rho_i\}$. However, the contrary implication may not be true.

To investigate quantum broadcasting, we can consider two cases. One is that the states to be broadcast are pure states. It is not difficult to find that the only way to broadcast a pure state $|\psi_i\rangle$ is to put the two systems in the product state $|\psi_i\rangle|\psi_i\rangle$, that is quantum pure states cloning.

For a detailed review on quantum cloning, we may refer to [2]. The unitarity and linearity of quantum physics lead to some impossibilities—the no-cloning theorem [3–5] and the no-deleting principle [6]. The linearity of quantum theory makes an unknown quantum state unable to be perfectly copied [3, 4] and deleted [6], and two nonorthogonal states are not allowed to be precisely cloned and deleted, as a result of the unitarity [5, 7, 8], that is, for nonorthogonal pure states $|\psi_1\rangle$ and $|\psi_2\rangle$, no physical operation in quantum mechanics can exactly achieve the transformation $|\psi_i\rangle \rightarrow |\psi_i\rangle|\psi_i\rangle (i = 1, 2)$. Recently, Jozsa [9], Horodecki *et al* [10] and Azuma *et al* [11] further clarified the no-cloning theorem and the no-deleting principle from the viewpoint of conservation of quantum information. No-cloning theorem has also been generalized to entangled states [12]. Remarkably, these restrictions provide a valuable resource in quantum cryptography [13], because they forbid an eavesdropper to gain information on the distributed secret key without producing errors.

We briefly recall some preliminaries regarding quantum cloning. In general, there are two kinds of cloners. One is the *universal quantum copying machine* first introduced by Bužek and Hillery [14], and this kind of machines is deterministic and does not need any information about the states to be cloned, so it is *state independent*. The other kind of cloners is *state dependent* [15, 16], since it needs some information about the states to be cloned. Furthermore, this kind of cloning machines may be divided into three fashions: the first one is probabilistic cloner proposed first by Duan and Guo [17, 18], and then studied by Chefles and Barnett [19] and Pati [20], that can clone linearly independent states with nonzero probabilities. The issue that the supplementary information is added in Duan and Guo's probabilistic cloning [17, 18] and Pati's *novel cloning machine* (NCM) [20] was investigated by Azuma *et al* [21] and Qiu [22, 23]. The second one is deterministic cloner first investigated by Bruß *et al* [24] and then by Chefles and Barnett [25]; the last one is hybrid cloner studied by Chefles and Barnett [19], that combine deterministic cloner with probabilistic cloner.

Quantum broadcasting is a kind of more general cloning, and the states to be broadcast can be mixed states. Quantum broadcasting is also more complicated than universal quantum copying. Corresponding to the *no-cloning theorem* [3, 4], Barnum *et al* [1] presented the *no-broadcasting theorem* and showed that noncommuting mixed states cannot be broadcast determinately. Recently, the no-broadcasting theorem has drawn much attention in the academic community. Chen *et al* [26] showed that there does not exist any universal quantum cloning machine that can broadcast an arbitrary mixed qubit with a constant fidelity. Barnum *et al* [27] developed the no-broadcasting theorem and presented a more general form of *no-broadcasting theorem*. Kalev *et al* [28] presented a general proof for the no-broadcasting theorem.

However, if the input copies are more than one, the *no-broadcasting theorem* may not hold. Indeed, D'Ariano *et al* [29] investigated optimal universal broadcasting for mixed states of qubits and showed that for four or more input copies, it is even possible to purify the input states while broadcasting, and they called this phenomenon as *superbroadcasting*. Fan *et al* [30] proposed a quantum broadcasting transformation that can broadcast $\rho_i \otimes \rho_i$ to $M (M \geqslant 2)$ copies, and they showed that the shrinking factor between the input and the output single qubits is the upper bound. In their another paper [31], the N to $M(M \geqslant N)$ universal quantum broadcasting of mixed states $\rho_i^{\otimes N}$ was proposed for qubit systems.

So far, there is only one kind of approximate broadcasting, namely, the *optimal broadcasting* (OB), which broadcasts noncommuting mixed states approximately. Corresponding to the probabilistic cloning of nonorthogonal pure states, does there exist the second kind of broadcasting—the *probabilistic broadcasting* for noncommuting mixed states? Furthermore, what is the condition for mixed sates to be probabilistically broadcast? As an example, let $|c_{11}\rangle = |0\rangle, |c_{12}\rangle = |1\rangle, |c_{21}\rangle = \frac{|1\rangle+|2\rangle}{\sqrt{2}}, |c_{22}\rangle = \frac{|0\rangle+|3\rangle}{\sqrt{2}}$, and

$$\rho_1 = p_{11}|c_{11}\rangle\langle c_{11}| + p_{12}|c_{12}\rangle\langle c_{12}|, \tag{2}$$

$$\rho_2 = p_{21}|c_{21}\rangle\langle c_{21}| + p_{22}|c_{22}\rangle\langle c_{22}|. \tag{3}$$

It is obvious that ρ_1 and ρ_2 are noncommuting. Thus, according to the no-broadcasting theorem [1], they cannot be exactly broadcast. A natural question is whether they can be probabilistically broadcast.

With a different fashion (but it is related to the standard broadcasting task), Piani *et al* [32] investigated the local broadcasting of multipartite quantum correlations. They divided the separable bipartite states into three fashions: general separable bipartite states, quantum-classical states and classical–classical states, and showed that *classical–classical* states are the only states that can be locally broadcast. Giovannetti and Holevo [33] presented a weaker version of broadcasting to broadcast an unknown input state into two subsystems which partially overlap, which was named as *quantum shared broadcasting* (QSB). They showed that QSB strongly depends upon the overlap among the subsystems and analyzed the condition for approximate shared broadcasting.

Another question is, besides *classical–classical* states, whether the other separable multipartite states can be locally broadcast with a certain success probability. We will partially address these issues in this paper.

In this paper, motivated by the probabilistic cloning of quantum states proposed by Duan and Guo [17, 18], we address a probabilistic way for broadcasting of noncommuting mixed states—*probabilistic broadcasting*, and we present a sufficient condition of PB for mixed states. We generalize the no-broadcasting theorem [1] to the probabilistic setting. Therefore, the case that commuting mixed states can be broadcast exactly may be thought of as a special instance of PB in which the success ratio is 1. Moreover, we present *probabilistic local broadcasting* (PLB) of separable bipartite states.

The remainder of the paper is organized as follows. In section 2, we review related basic definitions and the probabilistic cloning theorem, and then introduce the protocol of probabilistic broadcasting used in later sections. In section 3, we prove our main results regarding probabilistic broadcasting of mixed states. In section 4, we discuss probabilistic local broadcasting of separable bipartite states. Finally, in section 5, we summarize our results, mention some potential of applications and address a number of related issues for further consideration.

2. Preliminaries

First, let us review the definition of separable bipartite states and the notion of local broadcast that will be used in section 4.

Definition 1. *([32]) A bipartite state ρ_{AB} is called as (i) separable if it can be written as $\rho_{AB} = \sum_i p_i\sigma_i^A \otimes \sigma_i^B$, where $\{p_i\}$ is a probability distribution and each σ_i^X is a quantum state,* *and if non-separable, it is named as entangled; (ii) classical-quantum (CQ) if it can be written as* $\sum_i p_i|i\rangle\langle i|\otimes\sigma_i^B$*, where* $\{|i\rangle\}$ *is an orthonormal set,* $\{p_i\}$ *is a probability distribution and each* σ_i^B *is a quantum state; (iii) classical–classical (CC), if there are two orthonormal sets* $\{|i\rangle\}$ *and* $\{|j\rangle\}$ *such that* $\rho=\sum_{ij}p_{ij}|i\rangle\langle i|\otimes|j\rangle\langle j|$*, with* $\{p_{ij}\}$ *a joint probability distribution for the indices* (i,j)*.*

Definition 2. *([32]) The bipartite state* ρ_{AB} *is locally broadcastable (LB) if there exist local maps* $\xi_A : A\to AA', \xi_B : B\to BB'$ *such that* $\sigma_{AA'BB'}\equiv(\xi_A\otimes\xi_B)(\rho_{AB})$ *satisfying* $\mathrm{tr}_{AB}(\sigma_{AA'BB'})=\mathrm{tr}_{A'B'}(\sigma_{AA'BB'})=\rho$*.*

Then, considering the close relationship between probabilistic broadcasting and probabilistic cloning, it is necessary to recall Duan and Guo's probabilistic cloning theorem [18].

Theorem 1. *([18]) There exists a cloning machine*

$$U(|\psi_i\rangle|\Sigma\rangle|P\rangle)=\sqrt{r_i}|\psi_i\rangle|\psi_i\rangle|P\rangle+\sqrt{1-r_i}\left|\Psi_{abp}^{(i)}\right\rangle,\qquad i=1,2,\ldots,n,\tag{4}$$

if and only if the matrix $X-\sqrt{\Gamma}Y\sqrt{\Gamma}$ *is positive semidefinite, where* U *is a unitary operator,* $|P\rangle$ *is a normalized state,* $|\Sigma\rangle$ *is a blank state,* $|\Psi_{abp}^{(i)}\rangle$ *is a normalized state of the composite system ABP such that* $\left\langle P\middle|\Psi_{abp}^{(i)}\right\rangle=0(i=1,2,\ldots,n)$*,* $X=[\langle\psi_i|\psi_j\rangle]$*,* $Y=[\langle\psi_i|\psi_j\rangle^2]$*, and* $\Gamma=\mathrm{diag}(r_1,r_2,\ldots,r_n)$ *are* $n\times n$ *matrices.*

Besides the form stated above, the probabilistic cloning theorem can also be equivalently presented as the following form.

Theorem 2. *([18]) The state secretly chosen from the set* $\{|\psi_i\rangle\}$ *can be probabilistically cloned if and only if the states in the set* $\{|\psi_i\rangle\}$ *are linearly independent.*

Now, we consider *probabilistic broadcasting* of mixed states. We present the protocol as following.

In our protocol, there are three players, Alice, Bob and Victor. Alice has system A whose initial state, to be broadcast to Bob, is secretly chosen from the set $\{\rho_i\}$. Bob has system B whose initial state is a blank state Σ used to receive the state ρ_i. Victor has system P whose initial state is P used to probe whether broadcasting is successful or not. First, let the three particles pass through a special unitary gate U, such that

$$U(\rho_i\otimes\Sigma\otimes P)U^\dagger=r_i\widetilde{\rho_i}\otimes P+(1-r_i)\widetilde{\sigma}_{abp}^{(i)},\qquad i=1,2,\ldots,n,\tag{5}$$

where $\widetilde{\sigma}_{abp}^{(i)}$ is a density operator of the composite system ABP such that $\mathrm{tr}_P\big((I\otimes P)\widetilde{\sigma}_{abp}^{(i)}\big)=0$, and $\mathrm{tr}_A(\widetilde{\rho_i})=\mathrm{tr}_B(\widetilde{\rho_i})=\rho_i$. After broadcasting, by measuring Victor's system P with projectors $\{P,I-P\}$, we can judge whether the broadcasting succeeds or not. Thus, r_i is the success ratio of probabilistically broadcasting ρ_i.

This PB protocol can be directly extended to any more subsystems. Now, based on the protocol stated above, we are ready to investigate the problem of probabilistic broadcasting of mixed states.

3. Probabilistic broadcasting of mixed states

Throughout this paper, we consider the states in the set $\{\rho_i\,|\,i=1,2\}$ as the original mixed states to be broadcast. In the remainder of the paper, we simply write it as $\{\rho_i\}$. We define that $\Sigma\equiv|\Sigma\rangle\langle\Sigma|$ and $P\equiv|P\rangle\langle P|$, where $|\Sigma\rangle$ is a standard quantum state, and $|P\rangle$ is the probe state. r_i is the success ratio of probabilistic broadcasting of ρ_i, and χ_i denotes the rank of ρ_i.

Motivated by the probabilistic cloning of pure states proposed by Duan and Guo [17, 18], if we clone every item $|c_{ik}\rangle\langle c_{ik}|$ of the spectral decomposition of ρ_i with the same success ratio r_i, what will happen? Indeed, doing this means that we can broadcast ρ_i with the success ratio r_i. If there exist the same items in the spectral decompositions of ρ_1 and ρ_2, then the success ratios of broadcasting ρ_1 and ρ_2 should be equal.

Here, we should pay attention to the two different representations, $S = \{|c_{ik}\rangle | i = 1, 2; k = 1, 2, \ldots, \chi_i\}$ and $|c_{ik}\rangle$, $(i = 1, 2; k = 1, 2, \ldots, \chi_i)$. The front representation is a set, and we often simply write the front one as $\{|c_{ik}\rangle\}$ in case no confusion results. However, the latter one is not a set, which may include the same item. For example, if $|c_{11}\rangle = |0\rangle$, $|c_{12}\rangle = |2\rangle$, $|c_{21}\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$, $|c_{22}\rangle = |0\rangle$, then the front representation $\{|c_{ik}\rangle | i = 1, 2; k = 1, 2\}$ is equal to the set $\{|0\rangle, |2\rangle, \frac{|0\rangle+|1\rangle}{\sqrt{2}}\}$ which has three elements. To describe all the four states, we should use the latter representation denoted them as $|c_{ik}\rangle$, $(i = 1, 2; k = 1, 2)$.

Theorem 3. *If there exists spectral decomposition of* ρ_i,

$$\rho_i = \sum_k p_{ik}|c_{ik}\rangle\langle c_{ik}|, \qquad i = 1, 2, \tag{6}$$

satisfying that the states in the set of $S = \{|c_{ik}\rangle\}$ *are linearly independent, then* $\{\rho_i\}$ *can be probabilistically broadcast by the machine described by*

$$U(\rho_i \otimes \Sigma \otimes P)U^\dagger = r_i\widetilde{\rho_i} \otimes P + (1 - r_i)\widetilde{\sigma}^{(i)}_{abp}, \qquad i = 1, 2, \tag{7}$$

where r_i *is the success ratio of probabilistic broadcasting of* ρ_i, Σ *is the density operator of a standard quantum state,* P *is the density operator of the probe state,* $\widetilde{\sigma}^{(i)}_{abp}$ *is a density operator of the composite system ABP such that* $\mathrm{tr}_P\big((I \otimes P)\widetilde{\sigma}^{(i)}_{abp}\big) = 0$, *and* $\mathrm{tr}_A(\widetilde{\rho_i}) = \mathrm{tr}_B(\widetilde{\rho_i}) = \rho_i$.

Proof. Let $S_1 = \{|c_{1k}\rangle \,|\, k = 1, 2, \ldots, \chi_1\}$, $S_2 = \{|c_{2k}\rangle \,|\, k = 1, 2, \ldots, \chi_2\}$. Then $S = S_1 \bigcup S_2$. We consider two cases to prove this theorem:

(1) If $|S| = |S_1| + |S_2|$, then, since the states in the set $S = \{|c_{ik}\rangle\}$ are linearly independent, the matrices $X = [\langle c_{ik}|c_{jt}\rangle]$ and $Y = [\langle c_{ik}|c_{jt}\rangle^2]$ are positive definite. Let $\Gamma = \mathrm{diag}(r_1, r_1, \ldots, r_i, r_i, \ldots, r_m, r_m)$. For small enough $r_i > 0$, the matrix $X - \sqrt{\Gamma}Y\sqrt{\Gamma}$ is also positive definite. According to theorem 1, we know that there exists a machine to probabilistically clone the states in the sets S_1 and S_2 with probabilities r_1 and r_2, respectively, as follows:

$$U(|c_{ik}\rangle|\Sigma\rangle|P\rangle) = \sqrt{r_i}|c_{ik}\rangle|c_{ik}\rangle|P\rangle + \sqrt{1 - r_i}\big|\Psi^{(ik)}_{abp}\big\rangle,$$
$$i = 1, 2; \qquad k = 1, 2, \ldots, \chi_i, \tag{8}$$

where U is a unitary operator, $|\Sigma\rangle$ is a blank state, $\big|\Psi^{(ik)}_{abp}\big\rangle$ is a normalized state of the composite system ABP such that $\big\langle\Psi^{(ik)}_{abp}\big|\Psi^{(it)}_{abp}\big\rangle = 0$ and $\big\langle P\big|\Psi^{(ik)}_{abp}\big\rangle = 0$.
As a result, we have

$$U(|c_{ik}\rangle|\Sigma\rangle|P\rangle\langle c_{ik}|\langle\Sigma|\langle P|)U^\dagger = r_i|c_{ik}\rangle|c_{ik}\rangle|P\rangle\langle c_{ik}|\langle c_{ik}|\langle P| + (1 - r_i)\eta^{(ik)}_{abp}, \tag{9}$$

where we define $\eta^{(ik)}_{abp}$ as

$$\eta^{(ik)}_{abp} \equiv \frac{1}{1 - r_i}\Big(\sqrt{r_i(1 - r_i)}|c_{ik}\rangle|c_{ik}\rangle|P\rangle\big\langle\Psi^{(ik)}_{abp}\big| \\ + \sqrt{r_i(1 - r_i)}\big|\Psi^{(ik)}_{abp}\big\rangle\langle c_{ik}|\langle c_{ik}|\langle P\big| + (1 - r_i)\big|\Psi^{(ik)}_{abp}\big\rangle\big\langle\Psi^{(ik)}_{abp}\big|\Big), \tag{10}$$

such that $\mathrm{tr}_P\big((I \otimes P)\eta^{(ik)}_{abp}\big) = 0$.

Then, we have

$$U(\rho_i \otimes \Sigma \otimes P)U^\dagger = \sum_k p_{ik} U(|c_{ik}\rangle|\Sigma\rangle|P\rangle\langle c_{ik}|\langle\Sigma|\langle P|)U^\dagger \tag{11}$$

$$= \sum_k p_{ik}\big(r_i|c_{ik}\rangle|c_{ik}\rangle|P\rangle\langle c_{ik}|\langle c_{ik}|\langle P| + (1-r_i)\eta_{abp}^{(ik)}\big) \tag{12}$$

$$= r_i \sum_k p_{ik}(|c_{ik}\rangle|c_{ik}\rangle|P\rangle\langle c_{ik}|\langle c_{ik}|\langle P|) + (1-r_i)\sum_k p_{ik}\eta_{abp}^{(ik)} \tag{13}$$

$$= r_i\widetilde{\rho_i} \otimes P + (1-r_i)\widetilde{\sigma}_{abp}^{(i)}, \tag{14}$$

where $\widetilde{\rho_i} = \sum_k p_{ik}|c_{ik}\rangle|c_{ik}\rangle\langle c_{ik}|\langle c_{ik}|$ satisfying $\mathrm{tr}_A(\widetilde{\rho_i}) = \mathrm{tr}_B(\widetilde{\rho_i}) = \rho_i$, and

$$\widetilde{\sigma}_{abp}^{(i)} = \sum_k p_{ik}\eta_{abp}^{(ik)}, \tag{15}$$

satisfying that $\mathrm{tr}_P\big((I\otimes P)\widetilde{\sigma}_{abp}^{(i)}\big) = \mathrm{tr}_P\big((I\otimes P)\sum_k p_{ik}\eta_{abp}^{(ik)}\big) = \sum_k p_{ik}\,\mathrm{tr}_P\big((I\otimes P)\eta_{abp}^{(ik)}\big) = 0$. After U transformation, Victor plays a measurement using $\{P, I-P\}$. Then, the state of the composite system AB will be $\widetilde{\rho_i}$ with success probability r_i, and $\mathrm{tr}_P\big(I\otimes(I-P)\widetilde{\sigma}_{abp}^{(i)}\big)$ with failure probability $1-r_i$. So, we have a machine (7) to broadcast the mixed states ρ_1 and ρ_2 with probabilities r_1 and r_2, respectively.

(2) If $|S| < |S_1| + |S_2|$, then, because the states in the set $S = \{|c_{ik}\rangle\}$ are linearly independent, the matrixes $X = [\langle c_{ik}|c_{jt}\rangle]$ and $Y = [\langle c_{ik}|c_{jt}\rangle^2]$ are positive definite. Let $\Gamma = \mathrm{diag}(r, r, \ldots, r)$. For small enough $r > 0$, the matrix $X - \sqrt{\Gamma}Y\sqrt{\Gamma}$ is also positive definite. According to theorem 1, there exists a machine to probabilistically clone the states in the set S with the same probability r. Thus, we have a cloning machine to probabilistically clone the states in the sets S_1 and S_2 with the same probability r. The cloning machine is described as follows:

$$U'(|c_{ik}\rangle|\Sigma\rangle|P\rangle) = \sqrt{r}|c_{ik}\rangle|c_{ik}\rangle|P\rangle + \sqrt{1-r}\big|\Psi_{abp}^{\prime(ik)}\big\rangle,$$
$$i = 1, 2; \qquad k = 1, 2, \ldots, \chi_i, \tag{16}$$

where U' is a unitary operator, $|\Sigma\rangle$ is a blank state, $\big|\Psi_{abp}^{\prime(ik)}\big\rangle$ is a normalized state of the composite system ABP such that $\langle P\big|\Psi_{abp}^{\prime(ik)}\big\rangle = 0$.

This case can be thought of as a special case of the previous one, where $r_1 = r_2 = r$. Similar to the preceding case, by calculation, we also have a machine (7) to broadcast the mixed states ρ_1 and ρ_2 with probabilities r_1 and r_2, respectively, where $r_1 = r_2 = r$. □

Based on the above result, we can immediately answer the question that we present in section 1. Let $|c_{11}\rangle = |0\rangle$, $|c_{12}\rangle = |1\rangle$, $|c_{21}\rangle = \frac{|1\rangle+|2\rangle}{\sqrt{2}}$, $|c_{22}\rangle = \frac{|0\rangle+|3\rangle}{\sqrt{2}}$ and

$$\rho_1 = p_{11}|c_{11}\rangle\langle c_{11}| + p_{12}|c_{12}\rangle\langle c_{12}|, \tag{17}$$

$$\rho_2 = p_{21}|c_{21}\rangle\langle c_{21}| + p_{22}|c_{22}\rangle\langle c_{22}|. \tag{18}$$

Because the above two equations are the spectral decompositions of ρ_1 and ρ_2, respectively, and $\{|c_{ik}\rangle\}$ are linearly independent, we can conclude that $\{\rho_i\}$ can be probabilistically broadcast.

Remark. If the dimension of the eigenspace corresponding to one eigenvalue is greater than one, then the spectral decomposition of ρ_i may have many representation forms, namely, $\{|c_{ik}\rangle\}$ may be not exclusive. In the spectral decompositions of ρ_1 and ρ_2, if there exists $|c_{ik}\rangle = \sum_t \beta_t|c_{jt}\rangle$, where $\sum_t |\beta_t|^2 = 1$ and $|c_{jt}\rangle$ are in the same eigenspace corresponding to a eigenvalue of ρ_j, then we can consider another spectral decomposition of ρ_j in which there is the same item $|c_{ik}\rangle$. For example, suppose that

$$\rho_1 = \frac{1}{2}\frac{|0\rangle + |1\rangle}{\sqrt{2}}\frac{\langle 0| + \langle 1|}{\sqrt{2}} + \frac{1}{2}\frac{|0\rangle - |1\rangle}{\sqrt{2}}\frac{\langle 0| - \langle 1|}{\sqrt{2}}, \tag{19}$$

$$\rho_2 = p_{21}|0\rangle\langle 0| + p_{22}\frac{|1\rangle + |2\rangle}{\sqrt{2}}\frac{\langle 1| + \langle 2|}{\sqrt{2}}. \tag{20}$$

Because $\frac{|0\rangle+|1\rangle}{\sqrt{2}}, \frac{|0\rangle-|1\rangle}{\sqrt{2}}, |0\rangle, \frac{|1\rangle+|2\rangle}{\sqrt{2}}$ are linearly dependent, it seems that we cannot judge whether ρ_1 and ρ_2 can be probabilistically broadcast using our results. However, if we consider another spectral decomposition of ρ_1: $\rho_1 = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$, then we can see that there exists the same item $|0\rangle$ in the spectral decompositions of ρ_1 and ρ_2. We delete one of them, then $|0\rangle, |1\rangle, \frac{|1\rangle+|2\rangle}{\sqrt{2}}$ are linearly independent. By means of theorem 3, we can conclude that ρ_1 and ρ_2 can be probabilistically broadcast.

After we have known that a set of mixed states can be probabilistically broadcast, a natural question is what is the success ratio of PB of ρ_i. To answer the question, we have the following result.

Suppose that there exist spectral decompositions of ρ_i such as $\rho_i = \sum_k p_{ik}|c_{ik}\rangle\langle c_{ik}|$, $i = 1, 2$. Let $S = \{|c_{ik}\rangle \mid i = 1, 2; k = 1, 2, \ldots, \chi_i\}$, $S_1 = \{|c_{1k}\rangle \mid k = 1, 2, \ldots, \chi_1\}$ and $S_2 = \{|c_{2k}\rangle | k = 1, 2, \ldots, \chi_2\}$. Now, we define $X'^{(1)}$, $X'^{(2)}$ and Γ' as follows. Let

$$X'^{(1)} \equiv \begin{pmatrix} \langle c_{11}|c_{11}\rangle & \ldots & 0 & \langle c_{11}|c_{2t_1}\rangle & \cdots & \langle c_{11}|c_{2t_{|S|-|S_1|}}\rangle \\ \ldots & \ddots & \ldots & \ldots & \ddots & \ldots \\ 0 & \cdots & \langle c_{1|S_1|}|c_{1|S_1|}\rangle & \langle c_{1|S_1|}|c_{2t_1}\rangle & \cdots & \langle c_{1|S_1|}|c_{2t_{|S|-|S_1|}}\rangle \\ \langle c_{2t_1}|c_{11}\rangle & \cdots & \langle c_{2t_1}|c_{1|S_1|}\rangle & \langle c_{2t_1}|c_{2t_1}\rangle & \cdots & 0 \\ \ldots & \ddots & \ldots & \ldots & \ddots & \ldots \\ \langle c_{2t_{|S|-|S_1|}}|c_{11}\rangle & \cdots & \langle c_{2t_{|S|-|S_1|}}|c_{1|S_1|}\rangle & 0 & \cdots & \langle c_{2t_{|S|-|S_1|}}|c_{2t_{|S|-|S_1|}}\rangle \end{pmatrix}$$

and

$$X'^{(2)} \equiv \begin{pmatrix} \langle c_{11}|c_{11}\rangle^2 & \cdots & 0 & \langle c_{11}|c_{2t_1}\rangle^2 & \cdots & \langle c_{11}|c_{2t_{|S|-|S_1|}}\rangle^2 \\ \ldots & \ddots & \ldots & \ldots & \ddots & \ldots \\ 0 & \cdots & \langle c_{1|S_1|}|c_{1|S_1|}\rangle^2 & \langle c_{1|S_1|}|c_{2t_1}\rangle^2 & \cdots & \langle c_{1|S_1|}|c_{2t_{|S|-|S_1|}}\rangle^2 \\ \langle c_{2t_1}|c_{11}\rangle^2 & \cdots & \langle c_{2t_1}|c_{1|S_1|}\rangle^2 & \langle c_{2t_1}|c_{2t_1}\rangle^2 & \cdots & 0 \\ \ldots & \ddots & \ldots & \ldots & \ddots & \ldots \\ \langle c_{2t_{|S|-|S_1|}}|c_{11}\rangle^2 & \cdots & \langle c_{2t_{|S|-|S_1|}}|c_{1|S_1|}\rangle^2 & 0 & \cdots & \langle c_{2t_{|S|-|S_1|}}|c_{2t_{|S|-|S_1|}}\rangle^2 \end{pmatrix}$$

be two $|S| \times |S|$ positive definite matrixes, where $|c_{1k}\rangle \in S_1 (k = 1, 2, \ldots, |S_1|)$, $|c_{2t_j}\rangle \in S_2 - S_1 (j = 1, 2, \ldots, |S| - |S_1|)$. Γ' is a $|S| \times |S|$ diagonal efficiency matrix defined by $\Gamma' \equiv \mathrm{diag}(R_1, R_2)$, where $R_1 = \mathrm{diag}(r_1, r_1, \ldots, r_1)$ is a $|S_1| \times |S_1|$ diagonal positive definite matrix, and $R_2 = \mathrm{diag}(r_2, r_2, \ldots, r_2)$ is a $(|S| - |S_1|) \times (|S| - |S_1|)$ diagonal positive definite matrix. If $|S| < |S_1| + |S_2|$, namely, there exist the same items in the sets S_1 and S_2, then we let $r_1 = r_2$.

Theorem 4. *If the matrix* $X'^{(1)} - \sqrt{\Gamma'}X'^{(2)}\sqrt{\Gamma'}$ *is positive semidefinite, then* $\{\rho_i\}$ *can be probabilistically broadcast by the machine described by*

$$U(\rho_i \otimes \Sigma \otimes P)U^\dagger = r_i\widetilde{\rho_i} \otimes P + (1 - r_i)\widetilde{\sigma}_{abp}^{(i)}, \qquad i = 1, 2, \tag{21}$$

where r_i is the success ratio of probabilistic broadcasting of ρ_i, Σ is the density operator of a standard quantum state, P is the density operator of the probe state, $\widetilde{\sigma}_{abp}^{(i)}$ is a density operator of the composite system ABP such that $\mathrm{tr}_P\big((I\otimes P)\widetilde{\sigma}_{abp}^{(i)}\big)=0$, and $\mathrm{tr}_A(\widetilde{\rho}_i)=\mathrm{tr}_B(\widetilde{\rho}_i)=\rho_i$.

Proof. According to theorem 1, if the matrix $X'^{(1)}-\sqrt{\Gamma'}X'^{(2)}\sqrt{\Gamma'}$ is positive semidefinite, then there exists a cloning machine to clone the states $|c_{ik}\rangle$ in the set S. The machine is described as follows:

$$U(|c_{ik}\rangle|\Sigma\rangle|P\rangle)=\sqrt{r_i}|c_{ik}\rangle|c_{ik}\rangle|P\rangle+\sqrt{1-r_i}\big|\Psi_{abp}^{(ik)}\big\rangle,$$
$$|c_{ik}\rangle\in\{|c_{ik}\rangle|i=1,2;\qquad k=1,2,\ldots,\chi_i\},\tag{22}$$

where U is a unitary operator, $|\Sigma\rangle$ is a blank state, $\big|\Psi_{abp}^{(ik)}\big\rangle$ is a normalized state of the composite system ABP such that $\big\langle P\big|\Psi_{abp}^{(ik)}\big\rangle=0$.

According to theorem 2, we know that the states in the set $S=\{|c_{ik}\rangle\}$ are linearly independent. Therefore, according to theorem 3, we have the conclusion that $\{\rho_i\}$ can be probabilistically broadcast by the machine described by

$$U(\rho_i\otimes\Sigma\otimes P)U^\dagger=r_i\widetilde{\rho}_i\otimes P+(1-r_i)\widetilde{\sigma}_{abp}^{(i)},\qquad i=1,2,\tag{23}$$

where r_i is the success ratio of probabilistic broadcasting of ρ_i, Σ is the density operator of a standard quantum state, P is the density operator of the probe state, $\widetilde{\sigma}_{abp}^{(i)}$ is a density operator of the composite system ABP such that $\mathrm{tr}_P\big((I\otimes P)\widetilde{\sigma}_{abp}^{(i)}\big)=0$, and $\mathrm{tr}_A(\widetilde{\rho}_i)=\mathrm{tr}_B(\widetilde{\rho}_i)=\rho_i$. □

According to the results presented by Barnum *et al* [1], commuting mixed states can be determinately broadcasted. If ρ_1 and ρ_2 are commuting, there exists an orthonormal basis $\{|c_i\rangle\}$ such that both ρ_1 and ρ_2 are diagonal with respect to that basis. So, we can determinately clone every item $|c_i\rangle\langle c_i|$. Then, we can precisely broadcast ρ_1 and ρ_2. This case can be thought of as a special case of probabilistic broadcasting, where the success probability is 1.

Now, let us take two simple examples to understand our results for PB of mixed states.

Example 1. As we present in section 1, let $|c_{11}\rangle=|0\rangle$, $|c_{12}\rangle=|1\rangle$, $|c_{21}\rangle=\frac{|1\rangle+|2\rangle}{\sqrt{2}}$, $|c_{22}\rangle=\frac{|0\rangle+|3\rangle}{\sqrt{2}}$, and

$$\rho_1=p_{11}|c_{11}\rangle\langle c_{11}|+p_{12}|c_{12}\rangle\langle c_{12}|,\tag{24}$$

$$\rho_2=p_{21}|c_{21}\rangle\langle c_{21}|+p_{22}|c_{22}\rangle\langle c_{22}|.\tag{25}$$

The states in the set $\{|c_{ik}\rangle\}$ are linearly independent, and thus ρ_1 and ρ_2 can be probabilistically broadcast. There is not the same item in the spectral decompositions of ρ_1 and ρ_2. We suppose that the success probabilities of PB of ρ_1 and ρ_2 are r_1 and r_2, respectively. According to our results, we have

$$X'^{(1)}=\begin{pmatrix}1&0&0&\frac{1}{\sqrt{2}}\\0&1&\frac{1}{\sqrt{2}}&0\\0&\frac{1}{\sqrt{2}}&1&0\\\frac{1}{\sqrt{2}}&0&0&1\end{pmatrix}\tag{26}$$

and

$$\sqrt{\Gamma'}X'^{(2)}\sqrt{\Gamma'}=\begin{pmatrix}r_1&0&0&\frac12\sqrt{r_1r_2}\\0&r_1&\frac12\sqrt{r_1r_2}&0\\0&\frac12\sqrt{r_1r_2}&r_2&0\\\frac12\sqrt{r_1r_2}&0&0&r_2\end{pmatrix}.\tag{27}$$

By calculating, $X'^{(1)} - \sqrt{\Gamma'}X'^{(2)}\sqrt{\Gamma'}$ is positive semidefinite if and only if $(1 - r_1)(1 - r_2) - \left(\frac{1}{\sqrt{2}} - \frac{1}{2}\sqrt{r_1 r_2}\right)^2 \geqslant 0$. $r_1 = r_2 = 2 - \sqrt{2}$ satisfies the inequality $(1 - r_1)(1 - r_2) - \left(\frac{1}{\sqrt{2}} - \frac{1}{2}\sqrt{r_1 r_2}\right)^2 \geqslant 0$. Therefore, we can probabilistically broadcast $\{\rho_1, \rho_2\}$ with success ratio $r_1 = r_2 = 2 - \sqrt{2}$ using our protocol.

Example 2. Suppose that we have $|c_{11}\rangle = |0\rangle, |c_{12}\rangle = |2\rangle, |c_{21}\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}, |c_{22}\rangle = \frac{|0\rangle-|1\rangle}{\sqrt{2}}$ $|c_{23}\rangle = \frac{|2\rangle+|3\rangle}{\sqrt{2}}$, and

$$\rho_1 = p_{11}|c_{11}\rangle\langle c_{11}| + p_{12}|c_{12}\rangle\langle c_{12}|, \tag{28}$$

$$\rho_2 = p|c_{21}\rangle\langle c_{21}| + p|c_{22}\rangle\langle c_{22}| + (1 - 2p)|c_{23}\rangle\langle c_{23}|. \tag{29}$$

The states in the set $\{|c_{ik}\rangle\}$ are linearly dependent. However, because $|c_{11}\rangle = |c_{21}\rangle + |c_{22}\rangle$, furthermore, $|c_{21}\rangle$ and $|c_{22}\rangle$ are the states in the same eigenspace corresponding to the eigenvalue p, we can consider another decompositions of ρ_1 and ρ_2 such as

$$\rho_1 = p_{11}|0\rangle\langle 0| + p_{12}|2\rangle\langle 2|, \tag{30}$$

$$\rho_2 = p|0\rangle\langle 0| + p|1\rangle\langle 1| + (1 - 2p)\frac{|2\rangle + |3\rangle}{\sqrt{2}}\frac{\langle 2| + \langle 3|}{\sqrt{2}}. \tag{31}$$

Then the mixed states ρ_1, ρ_2 can be probabilistically broadcasted, since $|0\rangle, |1\rangle, |2\rangle$ and $\frac{|2\rangle+|3\rangle}{\sqrt{2}}$ are linearly independent. According to our results, we have

$$X'^{(1)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 1 \end{pmatrix}. \tag{32}$$

There exists the same item $|0\rangle$ in the spectral decompositions of ρ_1 and ρ_2, so the success probabilities of broadcasting ρ_1 and ρ_2 are the same. Assume that the success ratio is r, we have

$$\sqrt{\Gamma'}X'^{(2)}\sqrt{\Gamma'} = r[\langle c_i|c_j\rangle^2] = \begin{pmatrix} r & 0 & 0 & 0 \\ 0 & r & 0 & 0 \\ 0 & 0 & r & \frac{1}{2}r \\ 0 & 0 & \frac{1}{2}r & r \end{pmatrix}. \tag{33}$$

Then we have

$$X'^{(1)} - \sqrt{\Gamma'}X'^{(2)}\sqrt{\Gamma'} = \begin{pmatrix} 1-r & 0 & 0 & 0 \\ 0 & 1-r & 0 & 0 \\ 0 & 0 & 1-r & \frac{1}{\sqrt{2}} - \frac{1}{2}r \\ 0 & 0 & \frac{1}{\sqrt{2}} - \frac{1}{2}r & 1-r \end{pmatrix}. \tag{34}$$

$X'^{(1)} - \sqrt{\Gamma'}X'^{(2)}\sqrt{\Gamma'}$ is positive semidefinite if and only if $(1-r)^2\left(1-\frac{1}{\sqrt{2}}-\frac{1}{2}r\right)\left(1+\frac{1}{\sqrt{2}}-\frac{3}{2}r\right) \geqslant 0$. Thus, we get $r \leqslant 2 - \sqrt{2}$. That is to say, we can broadcast $\{\rho_1, \rho_2\}$ with success ratio no more than $2 - \sqrt{2}$ using our PB protocol.

4. Probabilistic local broadcasting of separable bipartite states

Piani *et al* [32] investigated local broadcasting of multipartite quantum correlations and showed that classical–classical are the only states that can be locally broadcast. Applying the results of previous sections, some general separable bipartite quantum states can also be LB with a certain success probability.

In the remainder of this paper, we consider the separable bipartite state $\rho_{AB} = \sum_i \sigma_i^A \otimes \sigma_i^B$ as the original mixed state to be broadcast. We define that $\Sigma^{A'B'} \equiv \Sigma^{A'} \otimes \Sigma^{B'} = |\Sigma^{A'}\rangle\langle\Sigma^{A'}| \otimes |\Sigma^{B'}\rangle\langle\Sigma^{B'}|$ and $P^{AB} \equiv P^A \otimes P^B = |P^A\rangle\langle P^A| \otimes |P^B\rangle\langle P^B|$, where $|\Sigma^X\rangle$ is a standard quantum state of system X, and $|P^X\rangle$ is the probe state of system X, X is chosen from A or B. r_X is the success ratio of probabilistic broadcasting of σ_i^X.

We present the protocol of *probabilistic local broadcasting* as following. There are two players, Alice and Bob. Alice has three systems A, A' and P^A. Bob has three systems B, B' and P^B. The initial state of the composite system AB, to be broadcast to the composite system $A'B'$, is secretly chosen from the set $\{\sigma_i^A \otimes \sigma_i^B\}$ with a probability distribution $\{p_i\}$. The initial state of the composite system $A'B'$ is a blank state $\Sigma^{A'B'}$. The composite system P^{AB} whose initial state is P^{AB} used to probe whether broadcasting is successful or not. Alice and Bob can locally broadcast systems A and B with success probabilities r_A and r_B, respectively. If both of them are successful, then the work is successful, otherwise, the work is failure. Specifically, let Alice's and Bob's systems pass through two special local unitary gates U_A and U_B, respectively, such that

$$(U_A \otimes U_B)(\rho_{AB} \otimes \Sigma_{A'B'} \otimes P^{AB})\big(U_A^\dagger \otimes U_B^\dagger\big) = r\,\widetilde{\rho_{AA'BB'}} \otimes P^{AB} + (1-r)\widetilde{\Omega}^{AA'BB'P^{AB}} \tag{35}$$

where $\widetilde{\rho_{AA'BB'}} \otimes P^{AB}$ is a density operator of the composite system $AA'BB'P^{AB}$ such that $\mathrm{tr}_{P^{AB}}((I \otimes P^{AB})\widetilde{\Omega}^{AA'BB'P^{AB}}) = 0$ and $\mathrm{tr}_{AB}(\widetilde{\rho_{AA'BB'}}) = \mathrm{tr}_{A'B'}(\widetilde{\rho_{AA'BB'}}) = \rho_i$. After local broadcasting, by measuring systems P^A and P^B with projectors $\{P^A, I - P^A\}$ and $\{P^B, I - P^B\}$, respectively, we can judge whether the broadcasting is successful or not.

Theorem 5. *Suppose $\rho_{AB} = \sum_i p_i \sigma_i^A \otimes \sigma_i^B$. If σ_i^X is the convex hull of linearly independent pure states $\{|\psi_{ik}^X\rangle\langle\psi_{ik}^X|\}$, that is to say $\sigma_i^X = \sum_k p_{ik}^X|\psi_{ik}^X\rangle\langle\psi_{ik}^X|$, then the bipartite state ρ_{AB} can be locally broadcast with a certain probability.*

Proof. According to theorem 1, we know that $\{|\psi_{ik}^X\rangle\}$ can be probabilistically cloned with the same success ratio r_X by a machine

$$U_X\big(|\psi_{ik}^X\rangle|\Sigma^{X'}\rangle|P^X\rangle\big) = \sqrt{r_X}|\psi_{ik}^X\rangle|\psi_{ik}^{X'}\rangle|P^X\rangle + \sqrt{1-r_X}|\Psi_{ik}^{XX'P^X}\rangle, \tag{36}$$

where U_X is a unitary operator, $|\Sigma^{X'}\rangle$ is a blank state, $|\Psi_{ik}^{XX'P^X}\rangle$ is a normalized state of the composite system $XX'P^X$ such that $\langle P^X|\Psi_{ik}^{XX'P^X}\rangle = 0$.

Therefore, we have

$$\begin{aligned} &U_X\big(|\psi_{ik}^X\rangle|\Sigma^{X'}\rangle|P^X\rangle\langle\psi_{ik}^X|\langle\Sigma^{X'}|\langle P^X|\big)U_X^\dagger \\ &\quad= r_X|\psi_{ik}^X\rangle|\psi_{ik}^{X'}\rangle|P^X\rangle\langle\psi_{ik}^X|\langle\psi_{ik}^{X'}|\langle P^X| + (1-r_X)\eta_{ik}^{XX'P^X}, \end{aligned} \tag{37}$$

where we define $\eta_k^{XX'P^X}$ as

$$\begin{aligned} \eta_{ik}^{XX'P^X} \equiv{}& \frac{1}{1-r_X}\big(\sqrt{r_X(1-r_X)}|\psi_{ik}^X\rangle|\psi_{ik}^{X'}\rangle|P^X\rangle\langle\Psi_{ik}^{XX'P^X}| \\ &+ \sqrt{r_X(1-r_X)}|\Psi_{ik}^{XX'P^X}\rangle\langle\psi_{ik}^X|\langle\psi_{ik}^{X'}|\langle P^X| + (1-r_X)|\Psi_{ik}^{XX'P^X}\rangle\langle\Psi_{ik}^{XX'P^X}|\big), \end{aligned} \tag{38}$$

such that $\mathrm{tr}_{P^X}((I \otimes P^X)\eta_{ik}^{XX'P^X}) = 0$.

Then, we have

$$U_X\big(\sigma_i^X \otimes \Sigma^{X'} \otimes P^X\big)U_X^\dagger = r_X \widetilde{\sigma}_i^{XX'} \otimes P^X + (1-r_X)\widetilde{\pi}_i^{XX'P^X}, \tag{39}$$

where r_X is the success ratio of probabilistic broadcasting of σ_i^X,

$$\widetilde{\sigma}_i^{XX'} = \sum_k p_{ik}^X \big|\psi_{ik}^X\big\rangle\big|\psi_{ik}^{X'}\big\rangle\big\langle\psi_{ik}^X\big|\big\langle\psi_{ik}^{X'}\big| \tag{40}$$

such that $\mathrm{tr}_X\big(\widetilde{\sigma}_i^{XX'}\big) = \mathrm{tr}_{X'}\big(\widetilde{\sigma}_i^{XX'}\big) = \sigma_i^X$, and

$$\widetilde{\pi}_i^{XX'P^X} = \sum_k p_{ik}^X \eta_{ik}^{XX'P^X} \tag{41}$$

satisfying $\mathrm{tr}_{P^X}\big((I \otimes P^X)\widetilde{\pi}_i^{XX'P^X}\big) = 0$.

Thus, there exist U_A and U_B such that

$$\begin{aligned}
&(U_A \otimes U_B)(\rho_{AB} \otimes \Sigma^{A'B'} \otimes P^{AB})\big(U_A^\dagger \otimes U_B^\dagger\big) \\
&\quad = \sum_i p_i\big[U_A\big(\sigma_i^A \otimes \Sigma^{A'} \otimes P^A\big)U_A^\dagger\big] \otimes \big[U_B\big(\sigma_i^B \otimes \Sigma^{B'} \otimes P^B\big)U_B^\dagger\big] && (42)\\
&\quad = \sum_i p_i\big[r_A\widetilde{\sigma}_i^{AA'} \otimes P^A + (1-r_A)\widetilde{\pi}_i^{AA'P^A}\big] \\
&\qquad \otimes \big[r_B\widetilde{\sigma}_i^{BB'} \otimes P^B + (1-r_B)\widetilde{\pi}_i^{BB'P^B}\big] && (43)\\
&\quad = r\widetilde{\rho_{AA'BB'}} \otimes P^{AB} + (1-r)\widetilde{\Omega}^{AA'BB'P^{AB}}, && (44)
\end{aligned}$$

where $r = r_A r_B$,

$$\begin{aligned}
\widetilde{\Omega}^{AA'BB'P^{AB}} = {} & \frac{1}{1-r_Ar_B}\sum_i p_i\big[r_A(1-r_B)\widetilde{\sigma}_i^{AA'} \otimes P^A \otimes \widetilde{\pi}_i^{BB'P^B} \\
& + (1-r_A)r_B\widetilde{\pi}_i^{AA'P^A} \otimes \widetilde{\sigma}_i^{BB'} \otimes P^B + (1-r_A)(1-r_B)\widetilde{\pi}_i^{AA'P^A} \otimes \widetilde{\pi}_i^{BB'P^B}\big],
\end{aligned} \tag{45}$$

such that $\mathrm{tr}_{P^{AB}}((I \otimes P^{AB})\widetilde{\Omega}^{AA'BB'P^{AB}}) = 0$, and $\widetilde{\rho_{AA'BB'}} = \sum_i p_i\widetilde{\sigma}_i^{AA'} \otimes \widetilde{\sigma}_i^{BB'}$ satisfying $\mathrm{tr}_{AB}(\widetilde{\rho_{AA'BB'}}) = \mathrm{tr}_{A'B'}(\widetilde{\rho_{AA'BB'}}) = \rho_{AB}$.

After broadcasting, by measuring systems P^A and P^B with projectors $\{P^A, I - P^A\}$ and $\{P^B, I - P^B\}$, respectively, we can judge whether the broadcasting is successful or not. Consequently, ρ_{AB} can be locally broadcast with a certain probability r. □

Classical–classical can be considered as a special case of separable bipartite states, where σ_i^X is the convex hull of orthogonal states $\{|i\rangle\langle i|\}$. Based on our result, it can be LB with success probability 1. Moreover, our result can be directly extended to separable multipartite states. As a consequence, to a certain extent, we generalize *the no-local-broadcasting theorem* for quantum correlations presented by Piani *et al* [32].

5. Concluding remarks

It is well known that the non-broadcasting theorem is a fundamental principle of quantum communication. As we are aware, OB is the only one method to broadcast noncommuting mixed states approximately.

In this paper, we have proposed a new manner for broadcasting noncommuting mixed states—PB. The initial state of the composite system ABP is $\rho_i \otimes \Sigma \otimes P$, and then let the three particles pass through a special unitary gate U, such that

$$U(\rho_i \otimes \Sigma \otimes P)U^\dagger = r_i\widetilde{\rho}_i \otimes P + (1-r_i)\widetilde{\sigma}_{abp}^{(i)}, \qquad i = 1, 2, \ldots, n, \tag{46}$$

where $\widetilde{\sigma}_{abp}^{(i)}$ is a density operator of the composite system ABP such that $\mathrm{tr}_P((I\otimes P)\widetilde{\sigma}_{abp}^{(i)}) = 0$, and $\mathrm{tr}_A(\widetilde{\rho}_i) = \mathrm{tr}_B(\widetilde{\rho}_i) = \rho_i$. After broadcasting, by measuring Victor's system P using projectors $\{P, I - P\}$, we can judge whether the broadcasting is successful or not. As a result, we will get the precise density operator ρ_i in each subsystem with success probability r_i. Furthermore, this PB protocol can be directly extended to arbitrarily more subsystems.

Besides this, we have proposed a sufficient condition for PB of mixed states. If $\rho_i = \sum_k p_{ik}|c_{ik}\rangle\langle c_{ik}|(i = 1, 2)$ and the states in the set $\{|c_{ik}\rangle\}$ are linearly independent, $\{\rho_i\}$ can be probabilistically broadcast. According to our conclusion, the case that commuting mixed states can be broadcast exactly can be thought of a special instance of PB where the success ratio is 1. Moreover, we have introduced PLB of separable bipartite states, and presented a sufficient condition for PLB of separable bipartite states.

PB may get a precise density operator in each separate system with a certain probability. We hope that our results would provide some useful ideas in preserving important quantum information, parallel storage of quantum information in a quantum computer, and quantum cryptography.

An interesting problem is what is the sufficient and necessary condition for PB, and another nature problem is what is the maximum success ratio of PB of mixed states. Moreover, the probabilistic cloning devices for mixed states are still worthy of further consideration. We would like to explore these questions in the future.

Acknowledgments

The authors would like to thank the anonymous referees and editor for their invaluable comments and suggestions that greatly helped to improve the quality of this paper. This work is supported by the National Natural Science Foundation (Nos. 60573006, 60873055), the Research Foundation for the Doctorial Program of Higher School of Ministry of Education (No. 20050558015), and Program for New Century Excellent Talents in University (NCET) of China.

References

[1] Barnum H, Caves C M, Fuchs C A, Jozsa R and Schumacher B 1996 *Phys. Rev. Lett.* **76** 2818
[2] Scarani V, Iblisdir S and Gisin N 2005 *Rev. Mod. Phys.* **77** 1225
[3] Wootters W K and Zurek W H 1982 *Nature* **299** 802
[4] Dieks D 1982 *Phys. Lett.* A **92** 271
[5] Yuen H P 1986 *Phys. Lett.* A **113** 405
[6] Pati A K and Braunstein S L 2000 *Nature* **404** 164
[7] D'Ariano G M and Yuen H P 1996 *Phys. Rev. Lett.* **76** 2832
[8] Qiu D W 2002 *Phys. Rev.* A **65** 052303
Qiu D W 2002 *Phys. Rev.* A **65** 052329
[9] Jozsa R 2004 *IBM J. Res. Dev.* **48** 79
[10] Horodecki M, Horodecki R, Sen De A and Sen U 2004 arXiv:quant-ph/0407038
[11] Azuma K, Koashi M, Katsura H and Imoto N 2006 arXiv:quant-ph/0604009
[12] Koashi M and Imoto N 1998 *Phys. Rev. Lett.* **81** 4264
[13] Gisin N, Ribordy G, Tittel W and Zbinden H 2002 *Rev. Mod. Phys.* **74** 145
[14] Bužek V and Hillery M 1996 *Phys. Rev.* A **54** 1844
[15] Vidal G, Jonathan D and Nielsen M A 2002 *Phys. Rev.* A **62** 012304
[16] Karimipour V and Rezakhani A T 2002 *Phys. Rev.* A **66** 052111
[17] Duan L M and Guo G C 1998 *Phys. Lett.* A **243** 261
[18] Duan L M and Guo G C 1998 *Phys. Rev. Lett.* **80** 4999
[19] Chefles A and Barnett S M 1998 *J. Phys. A: Math. Gen.* **31** 10097
[20] Pati A K 1999 *Phys. Rev. Lett.* **83** 2849

[21] Azuma K, Shimamura J, Koashi M and Imoto N 2005 *Phys. Rev.* A **72** 032335
[22] Qiu D W 2006 *J. Phys. A: Math. Gen.* **39** 5135 (also arXiv:quant-ph/0511058)
[23] Li L J and Qiu D W 2007 *Phys. Lett.* A **362** 143
[24] Bruß D, DiVincenzo D P, Ekert A, Fuchs C A, Macchiavello C and Smolin J A 1998 *Phys. Rev.* A **57** 2368
[25] Chefles A and Barnett S M 1999 *Phys. Rev.* A **60** 136
[26] Chen L and Chen Y 2007 *Phys. Rev.* A **75** 062322
[27] Barnum H, Barrett J, Leifer M and Wilce A 2007 *Phys. Rev. Lett.* **99** 240501
[28] Kalevv A and Hen I 2008 *Phys. Rev. Lett.* **100** 210502
[29] D'Ariano G M, Macchiavello C and Perinotti P 2005 *Phys. Rev. Lett.* **95** 060503
[30] Fan H, Liu B and Shi K 2006 arXiv:quant-ph/0601017v4
[31] Fang G F and Fan H 2007 *Phys. Rev.* A **76** 022323
[32] Piani M, Horodecki P and Horodecki R 2008 *Phys. Rev. Lett.* **100** 090502
[33] Giovannetti V and Holevo A S 2008 *Quantum Inform. Prog.* **7** 55